

# SYLLABUS FOR M. Phil. / Ph.D. (STATISTICS)

## ENTRANCE EXAMINATION

### Part II (Statistics)

Weightage 50%

#### 1. Univariate Distributions:

Laplace, Lognormal and Cauchy distributions. Idea of truncated distributions, Truncated Poisson and Normal distributions.

Univariate compound distribution: Contagious distributions: Neyman type-A, Poisson- Binomial and Poisson –Negative Binomial distribution, Univariate Power series distributions.

Non-central distributions: Non-central chi-square, t and F distribution.

Ordered statistics, their distributions and properties, distribution of Range.

#### 2. Multivariate Analysis:

Multivariate distributions: Multinomial distribution, Marginal and Conditional distributions, Characteristic function. Multivariate Normal distribution, Characteristic function, Marginal and conditional distributions, Distribution of linear function. Distribution of sample mean vector.

Distribution of sample generalized variance. Wishart Distribution: p.d.f of Wishart distribution, Properties of Wishart distribution, Additive property, Distribution of HWH', marginal distribution of  $W_{11}$ , distribution of  $h'wh/h'\Sigma h$ ,  $h'\Sigma^{-1}h/h'w^{-1}h$ , Characteristic function.

Null and Non-null distribution of sample correlation coefficient r. Definition of Multiple and partial correlation coefficients. Null distributions of sample multiple and partial correlation coefficients. Testing of  $H_0$ : (i)  $\rho = 0$ , (ii)  $\rho = \rho_0$  (iii)  $\rho_{1(2,p)} = 0$  (iv)  $\rho_{12,3,\dots,p} = 0$  (v)  $\rho_{12,3,\dots,p} = \rho_0$

Hotelling  $T^2$  statistic. Null distribution of  $T^2$ , application in tests on mean vector for one and two multivariate normal populations and in testing equality of the components of mean vector (Problem of symmetry).

Multivariate Analysis of variance (MANOVA): One-Way classification problem.

Classification Problem and Fisher's linear discriminant function, Probabilities of misclassification, Classification with more than Two multivariate normal populations.

Concept and application of (i) Factor analysis (ii) Principal Component analysis and (iii) Canonical Correlation analysis.

#### 3. SAMPLE SURVEY

Concept of population, sample, sampling frame and sampling design, sampling strategy, Simple random sampling, Probability Proportional to size sampling (with

and without replacement), Stratified sampling, Systematic sampling and Cluster sampling (with equal and unequal cluster sizes), Two-stage sampling and its generalization.

Use of auxiliary information at estimation stage: ratio, product, difference and regression estimators, Sen-Midzuno sampling scheme and unbiased ratio estimators, Hartlay & Ross unbiased ratio type estimators : Two phase sampling for ratio and regression estimators.

#### **4. Statistical Inference:**

##### **(i) Estimation Theory:**

Concept of Estimator and Estimate, Different measures of closeness of an estimator : Pitman's closeness. Some desirable properties of estimators: Definition of Unbiasedness and Biasedness, Definition of Consistent estimator, Theorem of derivation of Consistent estimator, Efficiency: Best linear combination of unbiased estimator, BAN estimators. Sufficient statistics.

Neyman factorization theorem for discrete case, Minimal sufficient statistics, complete sufficient statistics, Minimum variance unbiased estimation: Lower bound of variance of an unbiased estimator, Cramer-Rao inequality, Minimum variance bound unbiased estimators, condition of existence of Minimum Variance Bound Unbiased Estimator. Chapman-Robbins inequality, Bhattacharya inequality, Rao-Blackwell theorem. Lehmann-Scheffe theorem, One parameter family of exponential distribution, Concept of finding uniformly minimum variance unbiased estimator (UMVUE)

Maximum likelihood estimator and its properties. Method of maximum likelihood, other methods of estimation: Method of moments, Method of minimum chi-square, Method of modified minimum chi squares, Method of scoring, MLE for grouped data, Method of scoring, Location invariance and scale invariance estimator and parameter, Pitman estimators for location and scale parameters.

Confidence intervals: Methods of finding confidence interval, Large sample confidence intervals, confidence intervals for parameters of elementary distributions, confidence bounds of fixed length, Stein's two-stage procedure.

##### **(ii) Testing of Hypotheses:**

**Basic concepts of testing of hypotheses:** Statistical Hypotheses- Simple and Composite. Statistical tests, Critical region, randomized test, non-randomised test, Errors of Type I and Type II, Size and Power of a test

**MP and UMP tests:** Neyman-Pearson's Lemma & Generalized Neyman-Pearson's Lemma and its applications to find Most Powerful test and UMP tests for families of distributions admitting monotone likelihood ratio, two sided hypotheses

**Unbiasedness for testing of hypotheses:** Similar test, relationship with UMP unbiased test, UMP similar test and its application for one parameter exponential family, Similarity and completeness, tests with Neyman structure, UMP unbiased

tests for multi-parameter exponential families

**Concept of Invariance in testing of hypotheses:** Maximal invariant test, most powerful invariant test

**Concept of least favourable distribution and its use in testing of hypotheses**

**Likelihood ratio tests:** Likelihood ratio test for simple and composite hypotheses.

**Sequential testing of hypotheses:**

Wald's sequential probability ratio test (SPRT), Properties of SPRT, approximate bounds, OC and ASN functions, Efficiency of SPRT, Fundamental identity of sequential analysis and its use to obtain OC and ASN functions of SPRT.

**Non-Parametric Methods for Testing of hypotheses:**

Tests for one sample, Tests for two related samples, Tests for two independent samples, Tests for k related samples, Tests for k independent samples, Tests of significance of measures of correlation.

## **5. Linear Models:**

The general linear model: Gauss-Markoff set up, least squares, and generalized least squares, Normal equations and least squares estimates, estimation of linear parametric functions, variances and covariances of least squares estimates, estimation with correlated observations.

Least squares estimates with restrictions on parameters, simultaneous estimates of linear parametric functions, Canonical form of the linear hypothesis model and Error and Estimation spaces.

Estimation of scale parameter in the general linear model by quadratic functions. Necessary and sufficient conditions for (i) a quadratic form to be distributed as chi-square (ii) independence of a linear form and a quadratic form (iii) independence of two quadratic forms.

Cochran's theorem and its generalizations.

Tests of hypotheses regarding parameters of a general linear model, tests involving linear functions of parameters, tests of sub hypotheses.

## **6. STATISTICAL QUALITY CONTROL AND RELIABILITY**

**Quality Control :** Quality and Statistical Quality control concept. Control Charts for Measurements. Control Charts for Attributes. Acceptance sampling Plans for Attributes. Cusum Charts. Standard Plans for Attributes. Plan for Acceptance Sampling by Measurement.

Concept of Total Quality Management (TQM): Accelerating use of TQM. Service Quality vs Product Quality. Strategic Quality Planning. Strategy and Strategic Planning Process. Strategic Quality Management.

Organizing for TQM. Productivity and Quality. The Leverage of Productivity. Quality management Systems vs Technology. Basic Measures of Productivity.

The Cost of Quality. Cost of Quality Defined. Different views of Quality Costs.

Quality Costs and its Measurement.

Criteria for Quality Programs. ISO 9000 and onwards.

Basic concepts and distributions for product life, failure rate. Hazard function, Reliability function for Exponential, Normal, Lognormal, Weibull and Gamma Distributions. Analysis of Complete Data. Linear analysis and maximum likelihood analysis of censored data for exponential distribution only. Introduction of Bayes Methods in Reliability. Accelerated life testing. System reliability.

## **7. Economic and Business Statistics:**

**Index Numbers:** Price, Quantity and Value indices. Price Index Numbers: Construction, Uses, Limitations, Tests for index numbers, Chain Index Number. Consumer Price Index, Wholesale Price Index and Index of Industrial Production – Construction of index numbers and uses. Cost of Living Index Number and Various Official Index Numbers, HDI (Human Development Index)

### **Demand Analysis:**

Concept related to demand and supply, price elastic ties of demand and supply, Methods of determining demand and supply curves for cross section data and time series data, Leontief's method, Pigou's Method, Engels Curves, Pareto's Law of Income Distribution.

**Time Series Analysis:** Definition and importance of time series analysis. Components of a Time series. Different methods for determination of trend, Methods for elimination of seasonal components. Determination of cyclic components. Variate difference method and their merits and demerits.

Stationary Time series, Box-Jenkins Models, Introduction to Autoregressive (AR) Models, Moving Average (MA) Models, Mixed Autoregressive Moving Average (ARMA) Models. Autoregressive Integrated Moving Average (ARIMA) Models. Properties of these models. Forecasting Techniques, Seasonal ARIMA model, Introduction to conditional Heteroscedasticity model; Volatility models, ARCH model, GARCH model, properties, estimation and forecasting of these models.

## **8. ECONOMETRICS**

The nature and role of econometrics, Introduction to econometric models.

### **Single Equation methods:**

**OLS estimation :** Estimation, Prediction, and tests of hypotheses.

**Multicollinearity :** Detection and consequences, Ridge estimator.

**Generalized least square method:** Estimation and prediction.

**Heteroscedasticity:** Problem of heteroscedasticity, Consequence of heteroscedasticity, Tests for detecting the presence and nature of heteroscedasticity, Methods for handling heteroscedasticity.

Grouping of observations.

**Autocorrelation :** Introduction, its consequences and tests. The BLUE procedure.

**Dummy and Lagged variables.**

**Simultaneous Equation methods :** The Identification problem, Rank and Order conditions.

**Estimation methods:** Recursive systems, 2SLS Estimators, Limited Information estimators, K-class estimators, Introduction to 3SLS estimators, and Full Information ML method.

## 9. OPERATIONS RESEARCH

**Linear Programming:** Definition of linear programming problem (LPP), Formulation of LPP, Solution of LPP by Simplex Method (including Big-M and Two-phase method)

**Duality:** Definition of Dual Problem. Rules for converting any Primal into its Dual, Properties of Duality, Dual-Simplex Method

**Transportation Problem:** Definition of Transportation Problem (TP), Methods for getting basic feasible solution and Optimum solution to TP, Degeneracy in TP, Unbalanced TP.

**Assignment Problem:** Definition of Assignment Problem (AP), Algorithm for solving an AP, Unbalanced AP, Routing Problem

**Inventory Management Systems:** Definition, Costs involved in Inventory Problems, Classical EOQ Models without and with shortages, Multi-item Deterministic Models, Probabilistic Inventory Models, Inventory Models with Price Breaks

**Simulation:** Types of Simulation, Monte Carlo Simulation. Applications in different fields.

**Sensitivity Analysis:** Basic concepts, Changes in the coefficient of objective function, components of vector  $b$  and of Matrix  $A$ , Addition / Deletion of variable in the problem, Addition / Deletion of constraint in the problem.

**Integer Programming:** All and mixed integer programming (IPP) problems, Gomory's all-IPP algorithm, The branch and bound technique

**Replacement Theory:** Types of Replacement Problem, Replacement of Items that Deteriorate, Replacement of Items that fails completely and that of Staff

**PERT / CPM:** Basic concepts, Construction and Time Calculation of the Network, Determination of Float and of the Critical Path, Crashing a Project, Scheduling a Project, Resource Analysis and Allocation, Application of PERT/ CPM.

**Sequencing:** Definition, Notations and Assumptions, Solution of Sequencing problem. Problems with  $n$ -jobs and 2-machines,  $n$ -jobs and 3-machines, Problems with  $n$ -jobs and  $m$ -machines.

## 10. POPULATION STUDIES

The nature of demography, demographic view of population, techniques of population studies. Basic demographic measures, sex-ratio, child-women ratio, crude rates, specific rates.

Concepts of Life Tables, Assumptions related to life tables, The columns of life tables, Complete and Abridged life tables, Construction of life tables.

Infant Mortality, Neonatal mortality, Prenatal mortality, Maternal mortality, death rates, standardized death rates.

Crude Birth Rate (CBR), General fertility rate (GFR), Age specific fertility rate, Total fertility rate (TFR), Gross reproduction rate (GRR), Net reproduction rate (NRR).

Marriage rates, divorce rates, age pattern of marriage, types of migration, migration rates, migrant components, migrant streams, internal migration, international migration.

Introduction, Simple Birth and Death Process, Stationary population models, Stable population models, intrinsic rate of growth, intrinsic age distribution, Quasi stability.

Inter - censal and Post - censal estimates, population projections, mathematical methods, component methods, mortality basis for projections, fertility basis for projections, migration basis for projections.

Definition of Census and its features, Organizing the Census, methods of enumeration, Census in India, Indian Census in 1991 and 2001.

\*\*\*\*\*